The Math Circle at Canisius,
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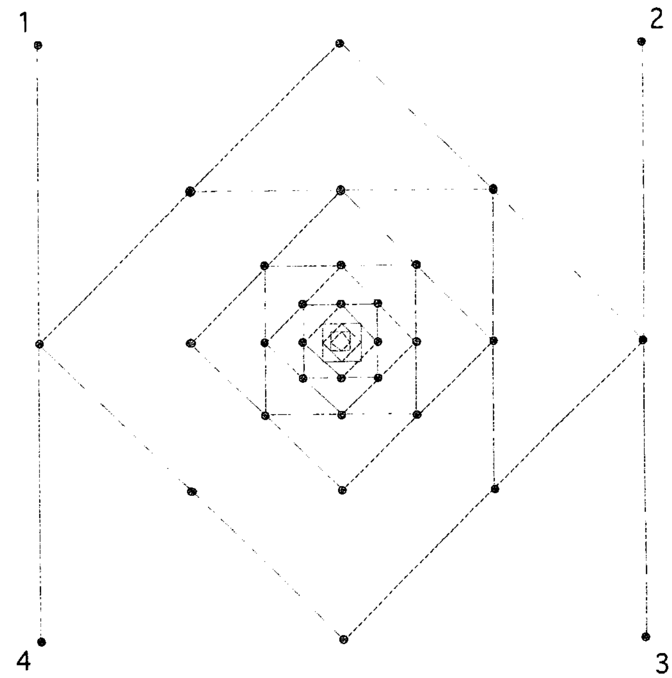


**A Cycle of Differences:**

Inspired by a math circle project
- Circle of Differences -
by Joshua Zucker,
at http://www.mathcircles.org/node/752

1. Take a (large) square, with a non-negative number at each corner.

At the midpoint of each side, write the absolute value of the difference between the numbers at its ends, forming a new square. Keep repeating ...

Try it with numbers 1, 2, 3, 4 at the corners (as shown). How many steps till the numbers get "stable"?

2. Suppose you start with just 0's and 1's at the corners.
What arrangement takes the largest number of steps before it gets stable?

3. Suppose you use only small numbers like 0, 1, 2 at the corners. Try for the largest number of steps.

4. Allowing big numbers, can you make the process last for more than 20 steps? How can you be sure?

5. What if you allow some fractions at corners; does the process eventually become stable?
Try some examples.

6. What happens if you start with 1, 2, $\pi$, 4 instead of 1, 2, 3, 4.

7. Can you explain why the process must always get stable if you start with whole numbers? With fractions? With other real numbers?

Now, what if we use shapes other than a square? Say, an $n$-sided regular polygon?

8. When $n = 3$, we may need to redefine our notion of "stable". Do all processes get stable in the same way?

9. In general, which values of $n$ fit with the original notion of "stable" for the square?

10. Can you establish a general upper bound on how long it takes for a given set of numbers to become stable, based on the largest number at a corner and the number $n$ of sides in the $n$-gon?