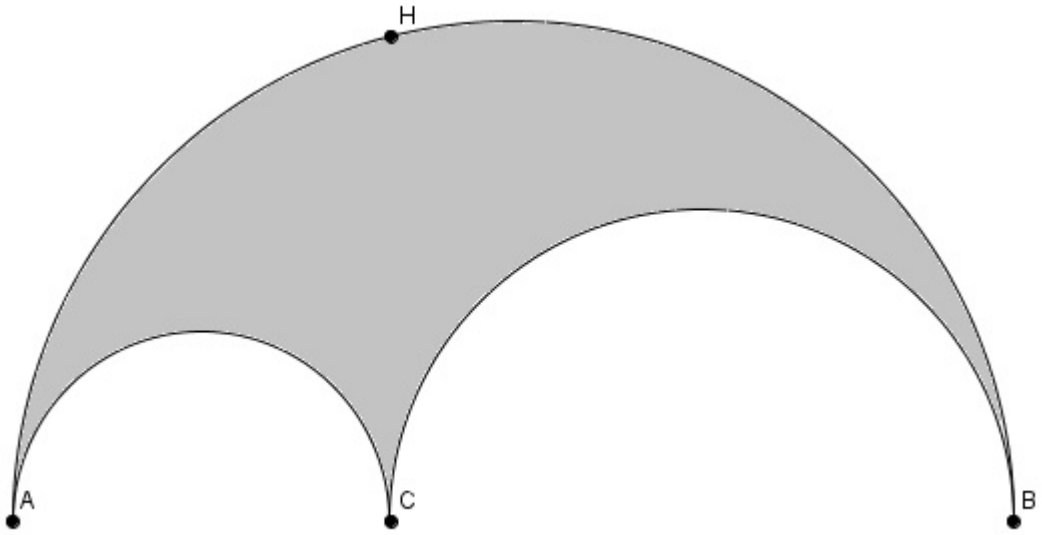
The $\alpha\rho\beta\eta\lambda o\sigma$

The Math Circle

November 13, 2012



1. Draw a few arbelos using the following steps:
   (a) Draw a semicircle with diameter on two points A and B.
   (b) Choose some point C on the diameter.
   (c) Bisect each segment $\overline{AC}$ at M_1 and $\overline{BC}$ at M_2.
   (d) On the same side of AB as part (a), draw semicircles with centers M_1 and M_2.

2. Are the lengths of the arcs forming the boundary of the arbelos related to one another?

3. Draw the perpendicular to $\overline{AB}$ intersecting the large arc of the arbelos at a point H. Now draw the triangle $\triangle M_1M_2H$. Can you find the area of this triangle? Are there any other triangles to be found in the arbelos? If so, which ones seem the most significant?

4. Is the area enclosed by the arbelos the same as the area of the circle whose diameter is $\overline{CH}$? Can you show why or why not?

5. Draw $\overline{AH}$ and $\overline{BH}$. These intersect the smaller arcs of the arbelos at X and Y respectively. Is XY tangent to the smaller arcs? How are $\overline{XY}$ and $\overline{CH}$ related?

6. Is there anything special about quadrilateral $XHYC$?

7. There is a circle which is tangent to all three of the semicircle arcs of the arbelos. This circle is called the arbelos' inscribed circle. What can you say about its area?

8. There are two circles which are tangent to one another, the large semicircle arc of the arbelos, and one of the smaller arcs. What can you say about the areas of these circles?

9. There is a circle tangent to XY and to the larger semicircle arc. How is the area of this circle related to the areas of the circles in the previous question?

10. There is a circle passing through the points of tangency between the smaller semicircle arcs and the inscribed circle, as well as point C. How does the area of this circle relate to the area of the circles in either of the previous two questions?