Brittany Zaccaria

Orders of Infinity- not all ∞ are the same

Firs t we recall some definitions from MAT 230 in order to get a better understanding. A **set** is any well-defined (its easily determined whether an object is in the set or not) collection or system of objects. A **subset** is given any two sets A and B, if a ϵ A implies a ϵ B for all a, we say is the set A is a subset of the set B. This leads to the idea of **power** (Cantor), the collection of all subsets of A. The **cardinality** of a set is simply the number of items in a set. Ex) Cardinality of {cat, dog} =2. Both cardinality and power is often used synonymously. A **denumerable set** is an infinite set equivalent to the set N, the set of all natural numbers.

When looking at the infinite sets, Cantor was motivated by the fact that elements from the set N are useful for counting the elements in sets having a finite number of elements, G. So he gave this question.

"Can the concept of natural numbers be generalized in such a manner that every set is assigned one of these generalized "numbers" for the "number of elements" in the set?"

He then went on to define the power of a set himself…

"The power of a given set A, is that general idea which remains with us when thinking of this set, we abstract from it all properties of its elements as well as from their order"

In MAT 230 it has already been proved that the Card ($\mathbb{Q}$) < Card ($\mathbb{R}$).

But is that true for all infinite sets will one always be bigger than the other? ---No.

Well we also know that $P(s)=2^s$

Ex) let A= {1,2,3}. P(A)= ({∅, {1}, {2}, {3}, {1,3}, {2,3}, {1,2}, {1,2,3})=8= 2^3.

This is useful for finite sets only however.

Definition: Two sets A and B have the **same cardinality (equivalent)**, written
A~B, if there exists a bijective function f : A → B. If no such bijective function exists, then A and B have **unequal cardinalities (not equivalent)**, that is lAl ~ lBl. When looking into infinite sets we can definitely see this to be true. For example,
A={1,2,3,…….}

B={2,4,6,,……}

Set B seems to have less power than the first due to the missing numbers in between, however ∃ f:A→B by f(n)= n/2, n∈ N so thus by definition actually have the same cardinality which Cantor represented using 0· (aleph naught) assigned to the class of all denumerable sets.