

## Game Theory

The games we will consider have certain things in common. These are:

- There is a finite set of *players* (who may be people, groups of people, or more abstract entities like computer programs or “nature” or “the house”).
- Each player has complete knowledge of the rules of the game.
- At different points in the game, each player has a range of choices or *moves*. This set of choices is finite.
- The game ends after a finite number of moves.
- After the game ends, each player receives a numerical *payoff*. This number may be negative, in which case it is interpreted as a loss of the absolute value of the number. For example, in a game like chess the payoff for winning might be +1, for losing -1, and for a draw 0.
- In some games, each player knows, at every point in the game, the entire previous history of the game. This is true of tic-tac-toe and backgammon, but not of bridge (because the cards dealt to other players are hidden). A game with this property is said to be *of perfect information*.

Note that not every game has these properties.

### Important Notes

- *Directed graph*: finite set of points, called *vertices*, together with a set of directed line segments, called *edges*, between some pair of distinct vertices.
- *Tree*: a directed graph  $T$  with a vertex  $r$ , called the *root*, such that  $r$  has no edges going into it and such that for every other vertex  $v$  of  $T$  there is a unique path from  $r$  to  $v$ .
- The tree of a two-player game can be turned into a matrix of the payoffs for each player, with player 1 as the row player and player 2 as the column player.
- Let  $M$  be an  $m \times n$  matrix. Then row  $i$  *dominates* row  $k$  if

$$m_{ij} \geq m_{kj} \text{ for all } j.$$

Also, column  $j$  *dominates* column  $l$  if

$$m_{ij} \leq m_{il} \text{ for all } i.$$

### Significance

Game theory has long been used by sociologists, economists and others interested in studying the evolution of cooperative behavior in nature and in human society. In the years after its invention in 1944, game theory was used by the U.S. Department of Defense to work on problems involving nuclear confrontation with the Soviet Union.