Eigenvalues and Additional Applications

Last talk, we discovered how incredibly important eigenvalues are to the operation of Google. Without them, Google's enormous matrix would be useless. In fact, without the power method (used for approximations when you are given a very large matrix), any attempt to solve the Google matrix would be futile. As such, in the next talk we will discuss:

Determining an Eigenvalue from an Eigenvector

If x is an eigenvector of a matrix A, then its corresponding eigenvalue is given by

$$\lambda = (Ax \cdot x)/(x \cdot x)$$

Dominant Eigenvalue

In order to use the power method, one must understand the definition of a **dominant eigenvalue**:

Let $\lambda_1, \lambda_2, \ldots,$ **and** λ_n be the eigenvalues of an $n \times n$ matrix A. λ_1 is called the **dominant eigenvalue** of A if

$|\lambda_1| > |\lambda_i|, i = 2, \ldots, n$

The eigenvectors corresponding to λ_1 are called **dominant eigenvectors** of A.

The Power Method

Assume that the matrix A has a dominant eigenvalue with corresponding dominant eigenvectors. Then choose an initial approximation of one of the dominant eigenvectors of A. This initial approximation must be a nonzero vector in R^n. Finally we form the sequence given by

$$x_1 = Ax_0$$

$$x_2 = Ax_1 = A(Ax_0) = A^2x_0$$

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$$x_k = Ax_{k-1} = A(A^{k-1}x_0) = A^kx_0$$