The Ford-Fulkerson Method of Maximum Flow Diagrams

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1. Background Information:

   The Ford-Fulkerson method is a way of finding the maximum flow in a directed graph given certain constraints. In this particular case, those constraints will be governed by the maximum capacity of edges along a path. Essentially, it takes a given path from the source to the sink and forces increased flow in the graph by modifying the original path and introducing alternate paths to get to the sink.

   There are many practical applications of the Ford-Fulkerson Method. In the field of ecology, it can be used to visually display the flow of energy in a food chain. It also has applications in infrastructure, namely plumbing and transportation systems. The most notable application is that of business. Shipping companies, such as FedEx and the US Postal Service, can use the Ford-Fulkerson Method to select the most efficient routes for deliveries. In a world where companies consistently seek to maximize efficiency and minimize cost, this concept is essential.

2. Definitions:
   - Directed Graph: a graph in which each edge has a given direction
   - Flow: the current value along a certain edge
   - Capacity: the maximum possible flow along an edge
   - Source: the vertex which produces flow
   - Sink: the vertex which consumes flow
   - Residual Paths: modified edges added to the original graph
   - Augmented Paths: the modified path from source to sink including residual paths
3. Properties of Maximum Flow Diagrams:
   - Capacity Constraints: $f(u,v) \leq c(u,v)$
     - f is the flow
     - c is the maximum capacity
     - u and v are two vertices, and the edge connects those two vertices
   - Skew Symmetry: $f(u,v) = -f(v,u)$
     - Going forward on an edge will yield a positive value.
     - Going backwards on an edge will yield a negative value
     - Used in residual paths
   - Flow Conservation: $\sum_{w \in v} f(u,w) = 0$ unless u = s or u = t
     - The flow into a vertex is equal to the flow out of a vertex, unless that vertex is the source or sink of the graph