The Math Circle at Canisius,
Tuesday March 4, 2014

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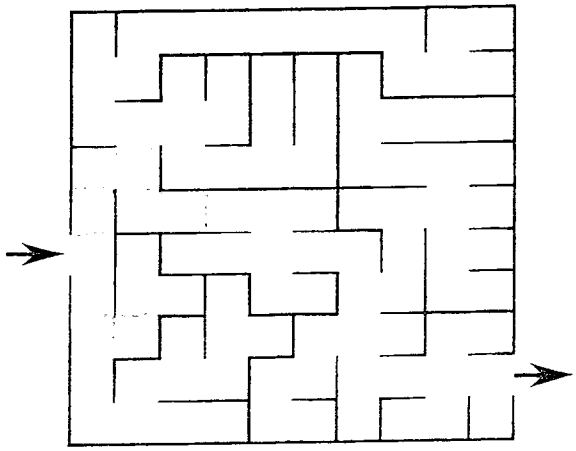


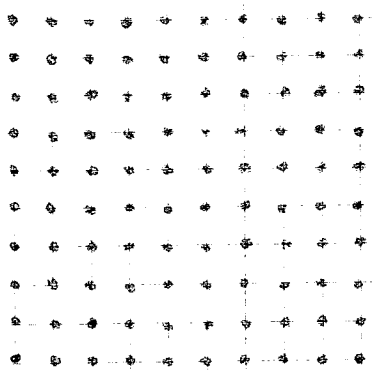
Building Maze-Trees:

1. Here is a maze. Is there a path from start to end in the maze? Find a shortest path, avoiding dead ends.
   OK, that was easy ...

2. How many steps in your path? Put a dot at the center of each square along the path, and count each connecting edge (from one dot to the next) as a step.

3. A pattern of dots and edges is called a **tree** when there is just one path from start to end. Usually, a maze has a tree of possible paths, with all dead-end paths shown as side branches. Draw the complete maze-tree for this maze; how many edges does it have? How many dots?

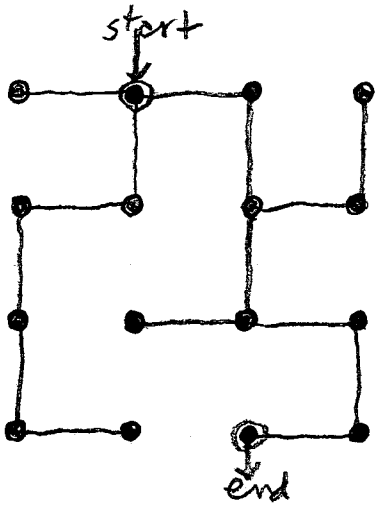
4. Draw a random-looking maze-tree connecting all the dots in the 10 by 10 grid below, with a start and an end dot. Can you guess at a relation between the number of dots and edges in a maze-tree?



5. Each dot in a maze-tree is met by 1 or 2 or 3 or 4 edges. How many of each type in your maze-tree?
   There is a theory that predicts the proportion of dots of each type in a **random** maze ...
   I learned about it in a recent book: *Probability on Trees and Networks*, by Russell Lyons.

What makes a good maze?

Let's experiment with trees
on a 4 by 4 square of dots,
with start/end dots circled.



6. Here is a sample maze-tree. Try to draw the walls for the maze?

7. How long is the shortest path from start to end?

8. A zombi might just follow the right-hand rule from start to end.
That will work, but how many steps will it take?

9. Suppose the zombi is left-handed; how many steps will it take?
Are the right-hand and left-hand answers related?

10. Experiment with other trees on a 4 by 4 square of dots ... What makes a good maze?