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MAT 380
Oct. 10, 2012

Basic Urn Models and the Pólya Urn Scheme

Urn models are popular because they simplify real problems into easy to understand probability questions; especially in fields of physics, communication theory, genetics, economics and computer science. The basic urn model starts off with many objects that you need to allocate to specified locations, the objects are represented as balls and the location(s) can be represented as urns.

Suppose Urn_q has j black balls and i red balls, what is the probability that the first ball you randomly select is black? This is a basic example of a question based on this model. There are many variations: having multiple urns, drawing multiple balls, with or without replacement, and so on.

Probability Background

In the simple example above, it's obvious that on the first draw P(black)= $^{j}/_{i+j}$ and P(red)= $^{i}/_{i+j}$. But here are some probability rules that are needed for more complicated problems:

$P(A \cup B) = P(A) + P(B)$, The probability of event A or event B (Only for disjoint situations: when both events cannot occur at the same time)

$P(A \cap B) = P(A)P(B)$, The probability of event A and event B (Only for independent events: when the occurrence of one event does not effect the other)

$P(A|B) = \frac{P(A \cap B)}{P(B)}$, The probability of event A given that event B already occurred, (this is called Conditional Probability: only works for dependent events.)

$P(A \cap B) = P(A|B)P(B)$, This is the Multiplication Rule, can also use P(A) instead of P(B).

Pólya Urn Scheme

This is a specific variation on the basic urn model, the Pólya Urn Scheme explores what happens when you draw a ball at random and it is replaced together with a certain number of balls of the same color. This is a popular model that represents how contagious diseases can spread. When a sample is taken (ball is drawn) from a population (urn) and the person is infected (say this is when the ball is red), then this increases the probability of other people being infected (so, it increases the probability that you'd draw a red ball).